

Eulerian graph

Definition:

A **graph** is called an **Eulerian Graph** if it contains an **Eulerian circuit**.

👉 An **Eulerian circuit (or Eulerian cycle)** is a closed trail (a path that starts and ends at the same vertex) that **uses every edge of the graph exactly once**.

Conditions for a Graph to be Eulerian:

For an **undirected graph**:

1. The graph must be **connected**.
2. Every vertex must have an **even degree** (number of edges incident to the vertex).

For a **directed graph**:

1. The graph must be **strongly connected**.
2. In-degree = Out-degree for every vertex.

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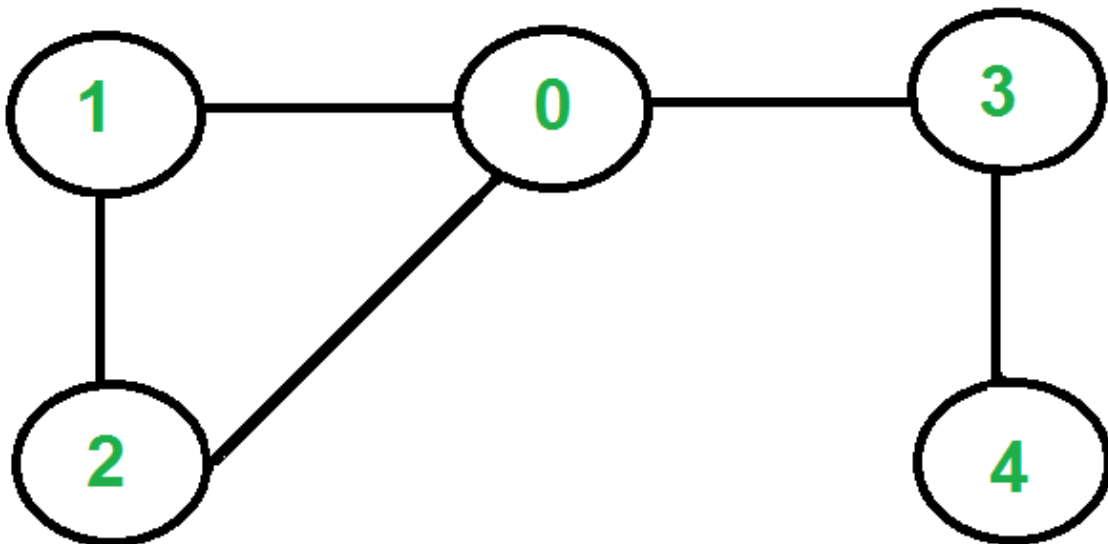
Eulerian Cycle

A graph has an Eulerian Cycle **if and only if** the below two conditions are **true**

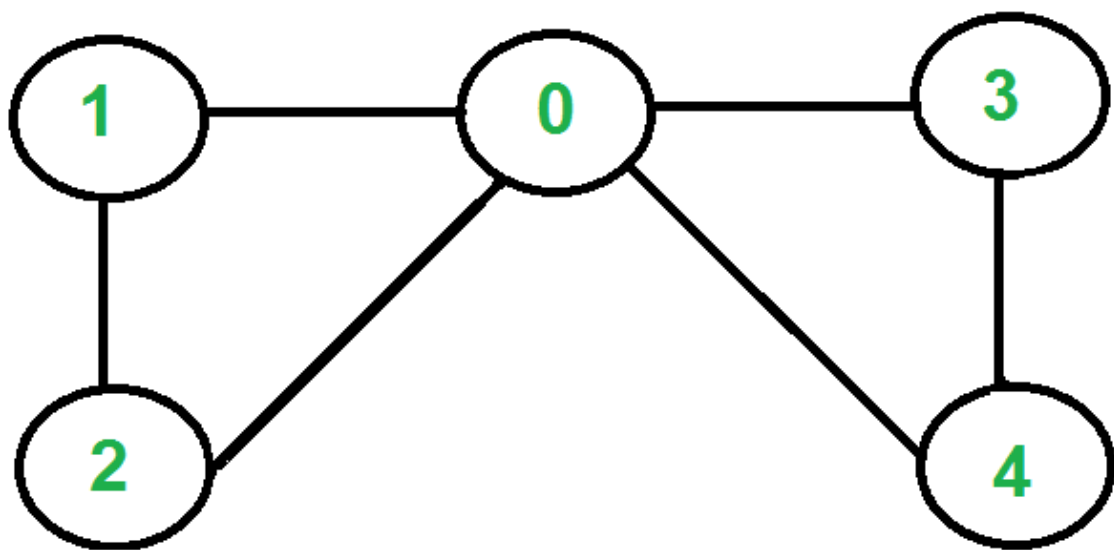
- All vertices with non-zero degree are part of a single connected component. (We ignore isolated vertices— those with zero degree — as they do not affect the cycle.)
- Every vertex in the graph has an even degree.
 - In an **Eulerian Path**, whenever we enter a vertex (except start and end), we must also leave it. So all **intermediate vertices** must have even degree.

- In an **Eulerian Cycle**, since we start and end at the same vertex, **every** vertex must have even degree. This ensures that every entry into a vertex can be paired with an exit.

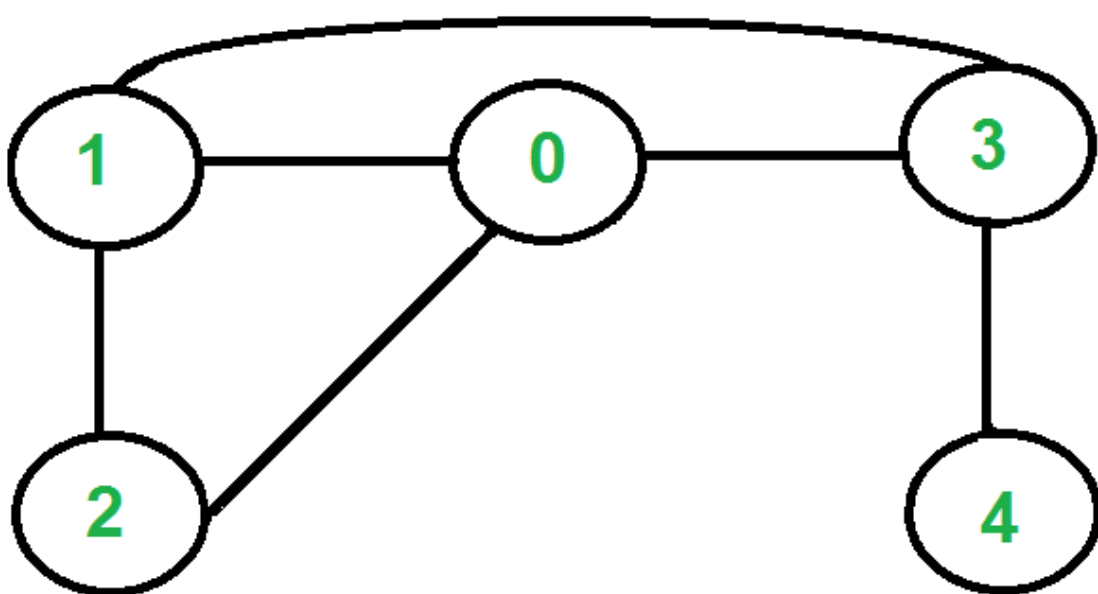
Examples:



The graph has Eulerian Paths, for example "4 3 0 1 2 0", but no Eulerian Cycle. Note that there are two vertices with odd degree (4 and 0)



The graph has Eulerian Cycles, for example "2 1 0 3 4 0 2"
Note that all vertices have even degree



The graph is not Eulerian. Note that there are four vertices with odd degree (0, 1, 3 and 4)